Code No. D-2322/N/AICTE

FACULTY OF ENGINEERING

B.E. (AICTE) I – Semester(Common for All Branches) (Main & Backlog)
Examinations, March / April 2022
Subject: Mathematics - I

Time: 3 Hours **Max. Marks: 70**

Note: (i) First question is compulsory and answer any four questions from the remaining six questions. Each Question carries 14 Marks.
(ii) Answer to each question must be written at one place only and in the same order as they occur in the question paper.
(iii) Missing data, if any, may be suitably assumed.

1.

(a) Discuss the convergence of the series $\sum \frac{1}{n^2}$.

(b) Obtain the fourth degree Taylor's polynomial approximation to $f(x) = e^{2x}$ about $x = 0$.

(c) Show that the following function $f(x, y)$ is continuous at the point (0,0).

$$f(x,y) = \begin{cases} \frac{2x(x^2-y^2)}{x^2+y^2}, & (x,y) \neq (0,0) \\ 0, & (x,y) = (0,0) \end{cases}.$$

(d) Evaluate the double integral $\iint_R e^{x^2} dx\, dy$, where the region R is given by $R: 2y \leq x \leq 2$ and $0 \leq y \leq 1$.

(e) Find the directional derivative of $f(x,y,z) = xy^2 + 4xyz + z^2$ at the point (1,2,3) in the direction of *3i+4j-5k*.

(f) If $\boldsymbol{r} = x\boldsymbol{i} + y\boldsymbol{j} + z\boldsymbol{k}$ and $r = |\boldsymbol{r}|$ show that $div\left(\frac{\boldsymbol{r}}{r^3}\right) = 0$.

2. (a) Examine the convergence or divergence of the following series: $\sum \frac{x^{n+1}}{(n+1)\sqrt{n}}$.

(b) Test the convergence of the series $\sum \frac{(-1)^{n-1}}{(2n-2)!}$.

3. (a) Obtain the Taylor's polynomial approximation of degree n to the function $f(x) = e^x$ about the point $x = 0$.

(b) Using Lagrange mean value theorem, show that $1 + x < e^x < 1 + xe^x$.

4. (a) If $f(x,y) = \tan^{-1}(xy)$, find an approximate value of $f(1.1, 0.8)$ using the Taylor's series (i) linear approximation and (ii) quadratic approximation.

(b) Find the shortest distance between the line $y = 10 - 2x$ and the ellipse $\frac{x^2}{4} + \frac{y^2}{9} = 1$.

5. (a) Evaluate the integral $\iiint_R (x^2 y) dx\, dy\, dz$, where the boundary R: $x^2 + y^2 \leq 1, 0 \leq z \leq 1$.

(b) Evaluate the integral $\iiint_R (2x - y - z) dx\, dy\, dz$, where the boundary R: $0 \leq x \leq 1, 0 \leq y \leq x^2, 0 \leq z \leq x + y$.

6. (a) Find the work done by the force $F = (x^2 - y^3)i + (x + y)j$ in moving a particle along the closed path C containing the curves $x + y = 0, x^2 + y^2 = 16$ and $y = x$ in the first and fourth quadrants.

   (b) Let D be the region bounded by the closed cylinder $x^2 + y^2 = 16, z = 0$ and $z = 4$. Verify the divergence theorem if $\mathbf{v} = 3x^2 i + 6y^2 j + zk$.

7. (a) Evaluate the surface integral $\iint_S F.n\, dA$ where $F = 6z\, i + 6j + 3yk$ and S is the portion of the plane $2x + 3y + 4z = 12$, which is in the first octant.

   (b) Evaluate the integral $\iint_S (\nabla \times \mathbf{v}).n\, dA$ by Stoke's theorem where $\mathbf{v} = (x^2 - y^2)i + (y^2 - x^2)j + zk$ and S is the portion of the surface $x^2 + y^2 - 2by + bx = 0, b$ constant, whose boundary lies in the x-y plane.